3-6 The Quadratic Formula and the Discriminant

Solve each equation by using the Quadratic Formula.

1. $x^2 + 12x - 9 = 0$

ANSWER:

$-6 \pm 3\sqrt{5}$

3. $4x^2 - 5x - 2 = 0$

ANSWER:

$\dfrac{5 \pm \sqrt{57}}{8}$

5. $10x^2 - 3 = 13x$
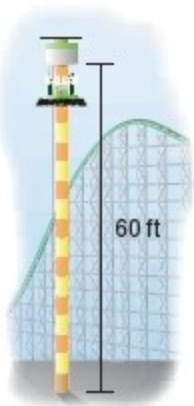
ANSWER:

$(1.5, -0.2)$

7. $-3x^2 + 4x = -8$

ANSWER:

$\dfrac{2 \pm 2\sqrt{7}}{3}$

9. **CCSS MODELING** An amusement park ride takes riders to the top of a tower and drops them at speeds reaching 80 feet per second. A function that models this ride is $h = -16t^2 - 64t - 60$, where h is the height in feet and t is the time in seconds. About how many seconds does it take for riders to drop from 60 feet to 0 feet?

ANSWER:
about 0.78 second

Complete parts *a* and *b* for each quadratic equation.

a. Find the value of the discriminant.

b. Describe the number and type of roots.

11. $2x^2 - 6x + 9 = 0$

ANSWER:
a. −36

b. 2 complex roots

13. $5x^2 + 2x + 4 = 0$

ANSWER:
a. −76

b. 2 complex roots

Solve each equation by using the Quadratic Formula.

15. $4x^2 - 6 = -12x$

ANSWER:

$$\frac{-3 \pm \sqrt{15}}{2}$$

17. $4x^2 - 9 = -7x - 4$

ANSWER:

$$\frac{-7 \pm \sqrt{129}}{8}$$

19. $12x^2 + 9x - 2 = -17$

ANSWER:

$$\frac{-3 \pm i\sqrt{71}}{8}$$

Complete parts *a–c* for each quadratic equation.
a. Find the value of the discriminant.
b. Describe the number and type of roots.
c. Find the exact solutions by using the Quadratic Formula.

21. $2x^2 + 3x - 3 = 0$

ANSWER:

a. 33

b. 2 irrational

c. $\dfrac{-3 \pm \sqrt{33}}{4}$

23. $6x^2 + 5x - 1 = 0$

    ANSWER:

    a. 49

    b. 2 rational

    c. $\dfrac{1}{6}, -1$

25. $3x^2 - 3x + 8 = 0$

    ANSWER:

    a. −87

    b. 2 complex

    c. $\dfrac{3 \pm i\sqrt{87}}{6}$

27. $-5x^2 + 4x + 1 = 0$

    ANSWER:

    a. 36

    b. 2 rational

    c. $1, -\dfrac{1}{5}$

29. $-3x^2 - 7x + 2 = 6$

    ANSWER:

    a. 1

    b. 2 rational

    c. $-1, -\dfrac{4}{3}$

31. $x^2 + 2x - 4 = -9$

    ANSWER:

    a. -16

    b. 2 complex

    c. $-1 \pm 2i$

33. **VIDEO GAMES** While Darnell is grounded his friend Jack brings him a video game. Darnell stands at his bedroom window, and Jack stands directly below the window. If Jack tosses a game cartridge to Darnell with an initial velocity of 35 feet per second, an equation for the height h feet of the cartridge after t seconds is $h = -16t^2 + 35t + 5$.

    a. If the window is 25 feet above the ground, will Darnell have 0, 1, or 2 chances to catch the video game cartridge?

    b. If Darnell is unable to catch the video game cartridge, when will it hit the ground?

    ANSWER:

    a. 0

    b. about 2.3 seconds

Complete parts a–c for each quadratic equation.
a. Find the value of the discriminant.
b. Describe the number and type of roots.
c. Find the exact solutions by using the Quadratic Formula.

35. $5x^2 + 8x = 0$

ANSWER:

a. 64

b. 2 rational

c. $0, -\dfrac{8}{5}$

37. $4x - 3 = -12x^2$

ANSWER:

a. 160

b. 2 irrational

c. $\dfrac{-1 \pm \sqrt{10}}{6}$

39. $0.6x^2 + 1.4x = 4.8$

ANSWER:

a. 13.48
b. 2 irrational
c. $\dfrac{-0.7 \pm \sqrt{3.37}}{0.6}$

41. **SMOKING** A decrease in smoking in the United States has resulted in lower death rates caused by lung cancer. The number of deaths per 100,000 people y can be approximated by $y = -0.26x^2 - 0.55x + 91.81$, where x represents the number of years after 2000.

a. Calculate the number of deaths per 100,000 people for 2015 and 2017.

b. Use the Quadratic Formula to solve for x when $y = 50$.

c. According to the quadratic function, when will the death rate be 0 per 100,000? Do you think that this prediction is reasonable? Why or why not?

Year	Deaths per 100,000
2000	91.8
2002	89.7
2004	85.5
2010	60.3
2015	?
2017	?

ANSWER:

a. 25.1, 7.3

b. 11.7

c. 2018; Sample answer: no; the death rate from cancer will never be 0 unless a cure is found. If and when a cure will be found cannot be predicted.

43. **CCSS CRITIQUE** Tama and Jonathan are determining the number of solutions of $3x^2 - 5x = 7$. Is either of them correct? Explain your reasoning.

Tama

$3x^2 - 5x = 7$

$b^2 - 4ac = (-5)^2 - 4(3)(7)$

$= -59$

Since the discriminant is negative, there are no real solutions.

Jonathan

$3x^2 - 5x = 7$

$3x^2 - 5x - 7 = 0$

$b^2 - 4ac = (-5)^2 - 4(3)(-7)$

$= 109$

Since the discriminant is positive, there are two real roots.

ANSWER:

Jonathan is correct; you must first write the equation in the form $ax^2 + bx + c = 0$ to determine the values of a, b, and c. Therefore, the value of c is -7, not 7.

45. **REASONING** Determine whether each statement is *sometimes, always,* or *never* true. Explain your reasoning.

a. In a quadratic equation in standard form, if a and c are different signs, then the solutions will be real.

b. If the discriminant of a quadratic equation is greater than 1, the two roots are real irrational numbers.

ANSWER:

a. Sample answer: Always; when a and c are opposite signs, then ac will always be negative and $-4ac$ will always be positive. Since b^2 will also always be positive, then $b^2 - 4ac$ represents the addition of two positive values, which will never be negative. Hence, the discriminant can never be negative and the solutions can never be imaginary.

b. Sample answer: Sometimes; the roots will only be irrational if $b^2 - 4ac$ is not a perfect square.

47. **CHALLENGE** Find the value(s) of m in the quadratic equation $x^2 + x + m + 1 = 0$ such that it has one solution.

    ANSWER:
    −0.75

49. A company determined that its monthly profit P is given by $P = -8x^2 + 165x - 100$, where x is the selling price for each unit of product. Which of the following is the best estimate of the maximum price per unit that the company can charge without losing money?

    A $10
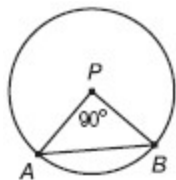
    B $20

    C $30

    D $40

    ANSWER:
    B

51. **SHORT RESPONSE** In the figure below, P is the center of the circle with radius 15 inches. What is the area of $\triangle APB$?

    ANSWER:
    112.5 in^2

    Find the value of c that makes each trinomial a perfect square. Then write the trinomial as a perfect square.

53. $x^2 + 13x + c$

    ANSWER:
    42.25; $(x + 6.5)^2$

55. $x^2 + \dfrac{4}{5}x + c$

ANSWER:

$\dfrac{4}{25}; \left(x + \dfrac{2}{5}\right)^2$
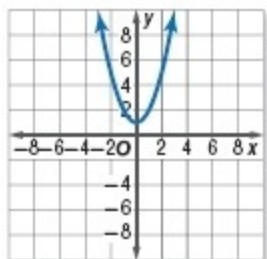
Simplify.

57. $\sqrt{-16}$

ANSWER:

$4i$

59. **PILOT TRAINING** Evita is training for her pilot's license. Flight instruction costs \$105 per hour, and the simulator costs \$45 per hour. She spent 4 more hours in airplane training than in the simulator. If Evita spent \$3870, how much time did she spend training in an airplane and in a simulator?

ANSWER:

27 hours of flight instruction and 23 hours in the simulator
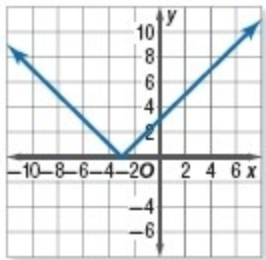
Write an equation for each graph.

61.

ANSWER:

$y = x^2 + 1$

63.

ANSWER:

$y = |x + 3|$