State the property that justifies each statement.

1. If \( m\angle 1 = m\angle 2 \) and \( m\angle 2 = m\angle 3 \), then \( m\angle 1 = m\angle 3 \).
   
   **ANSWER:**
   Trans. Prop.

3. If \( 5 = x \), then \( x = 5 \).
   
   **ANSWER:**
   Sym. Prop.

5. Complete the following proof.
   
   **ANSWER:**
   a. Given
   b. Multiplicative Property of Equality
   c. \( y + 2 = 9 \); Substitution
   
   **PROOF** Write a two-column proof to verify each conjecture.

7. If \( AB \cong CD \), then \( x = 7 \).
   
   **ANSWER:**
   Given: \( AB \cong CD \)
   Prove: \( x = 7 \)
   Proof:
   
   **Statements (Reasons)**
   1. \( AB \cong CD \) (Given)
   2. \( AB = CD \) (Def. of congruent segments)
   3. \( 4x - 6 = 22 \) (Subs. Prop.)
   4. \( 4x = 28 \) (Add. Prop.)
   5. \( x = 7 \) (Div. Prop.)
5-2 Algebraic Proof

State the property that justifies each statement.
9. If \(a + 10 = 20\), then \(a = 10\)

\textit{ANSWER:}
Subt. Prop.

11. If \(4x - 5 = x + 12\), then \(4x = x + 17\).

\textit{ANSWER:}
Add. Prop.

State the property that justifies each statement.
13. If \(5(x + 7) = -3\), then \(5x + 35 = -3\).

\textit{ANSWER:}
Dist. Prop.

15. If \(AB = BC\) and \(BC = CD\), then \(AB = CD\).

\textit{ANSWER:}
Trans. Prop.

CCSS ARGUMENTS Complete each proof.

17. Given: \(\frac{8 - 3x}{4} = 32\)

Prove: \(x = -40\)

\textbf{Proof:}

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. (\frac{8 - 3x}{4} = 32)</td>
<td>a. Given</td>
</tr>
<tr>
<td>b. (4 \left( \frac{8 - 3x}{4} \right) = 4(32))</td>
<td>b. ?</td>
</tr>
<tr>
<td>c. (8 - 3x = 128)</td>
<td>c. ?</td>
</tr>
<tr>
<td>d. ()</td>
<td>d. Subtraction Property</td>
</tr>
<tr>
<td>e. (x = -40)</td>
<td>e. ?</td>
</tr>
</tbody>
</table>

\textit{ANSWER:}
b. Multiplication Property of Equality
c. Substitution
d. \(-3x = 120\)
e. Division Property of Equality
5-2 Algebraic Proof

**PROOF** Write a two-column proof to verify each conjecture.

19. If \(-\frac{1}{3}n = 12\), then \(n = -36\)

**ANSWER:**

\[
\begin{align*}
\text{Given:} & \quad -\frac{1}{3}n = 12 \\
\text{Prove:} & \quad n = -36 \\
\text{Proof:} & \\
\text{Statements (Reasons)} & \\
1. & \quad -\frac{1}{3}n = 12 \text{ (Given)} \\
2. & \quad -3\left(-\frac{1}{3}n\right) = -3(12) \text{ (Mult. Prop.)} \\
3. & \quad n = -36 \text{ (Substitution)}
\end{align*}
\]
5-2 Algebraic Proof

21. **SCIENCE** Acceleration \( a \) in feet per second squared, distance traveled \( d \) in feet, velocity \( v \) in feet per second, and time \( t \) in seconds are related in the formula \( d = vt + \frac{1}{2}at^2 \).

a. Prove that if the values for distance, velocity, and time are known, then the acceleration of an object can be calculated using the formula \( a = \frac{2d - 2vt}{t^2} \).

b. If an object travels 2850 feet in 30 seconds with an initial velocity of 50 feet per second, what is the acceleration of the object? What property justifies your calculation?

**ANSWER:**

a. 

**Given:** \( d = vt + \frac{1}{2}at^2 \)

**Prove:** \( \frac{2d - 2vt}{t^2} = a \)

**Proof:**

<table>
<thead>
<tr>
<th>Statements (Reasons)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( d = vt + \frac{1}{2}at^2 ) (Given)</td>
</tr>
<tr>
<td>2. ( d - vt = vt - vt + \frac{1}{2}at^2 ) (Subt. Prop.)</td>
</tr>
<tr>
<td>3. ( d - vt = \frac{1}{2}at^2 ) (Subs.)</td>
</tr>
<tr>
<td>4. ( 2(d - vt) = 2\left(\frac{1}{2}at^2\right) ) (Mult. Prop.)</td>
</tr>
<tr>
<td>5. ( 2(d - vt) = at^2 ) (Subs.)</td>
</tr>
<tr>
<td>6. ( 2d - 2vt = at^2 ) (Dist. Prop.)</td>
</tr>
<tr>
<td>7. ( \frac{2d - 2vt}{t^2} = \frac{at^2}{t^2} ) (Div. Prop.)</td>
</tr>
<tr>
<td>8. ( \frac{2d - 2vt}{t^2} = a ) (Subs.)</td>
</tr>
<tr>
<td>9. ( a = \frac{2d - 2vt}{t^2} ) (Sym. Prop.)</td>
</tr>
</tbody>
</table>

b. 3 ft/ sec²;
5-2 Algebraic Proof

PROOF Write a two-column proof.

23. If $DF \cong EG$, then $x = 10$.

![Diagram with segments labeled 11 and 2x-9]

**ANSWER:**

Given: $DF \cong EG$
Prove: $x = 10$
Proof:

<table>
<thead>
<tr>
<th>Statements (Reasons)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $DF \cong EG$ (Given)</td>
</tr>
<tr>
<td>2. $DF = EG$ (Def. of $\cong$ segs)</td>
</tr>
<tr>
<td>3. $11 = 2x - 9$ (Subs.)</td>
</tr>
<tr>
<td>4. $20 = 2x$ (Add. Prop.)</td>
</tr>
<tr>
<td>5. $10 = x$ (Div. Prop.)</td>
</tr>
<tr>
<td>6. $x = 10$ (Symm. Prop.)</td>
</tr>
</tbody>
</table>

25. If $\angle Y \cong \angle Z$, then $x = 100$.

![Diagram with angles labeled (2x-90)° and (x+10)°]

**ANSWER:**

Given: $\angle Y \cong \angle Z$
Prove: $x = 100$
Proof:

<table>
<thead>
<tr>
<th>Statements (Reasons)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $\angle Y \cong \angle Z$ (Given)</td>
</tr>
<tr>
<td>2. $m\angle Y = m\angle Z$ (Def. of $\cong \angle$)</td>
</tr>
<tr>
<td>3. $x + 10 = 2x - 90$ (Subs.)</td>
</tr>
<tr>
<td>4. $10 = x - 90$ (Subt. Prop.)</td>
</tr>
<tr>
<td>5. $100 = x$ (Add. Prop.)</td>
</tr>
<tr>
<td>6. $x = 100$ (Sym. Prop.)</td>
</tr>
</tbody>
</table>
27. **ELECTRICITY** The voltage \( V \) of a circuit can be calculated using the formula \( V = \frac{P}{I} \), where \( P \) is the power and \( I \) is the current of the circuit.

a. Write a proof to show that when the power is constant, the voltage is halved when the current is doubled.

b. Write a proof to show that when the current is constant, the voltage is doubled when the power is doubled.

**Answer:**

**a.** Given: \( V = \frac{P}{I} \)

Prove: \( \frac{V}{2} = \frac{P}{2I} \)

**Proof:**

1. \( V = \frac{P}{I} \) (Given)

2. \( \frac{1}{2} \cdot V = \frac{1}{2} \cdot \frac{P}{I} \) (Mult. Prop.)

3. \( \frac{V}{2} = \frac{P}{2I} \) (Mult. Prop.)

**b.** Given: \( V = \frac{P}{I} \)

Prove: \( 2V = \frac{2P}{I} \)

**Proof:**

1. \( V = \frac{P}{I} \) (Given)

2. \( 2 \cdot V = 2 \cdot \frac{P}{I} \) (Mult. Prop.)

3. \( 2V = \frac{2P}{I} \) (Mult. Prop.)
29. **PYTHAGOREAN THEOREM** The Pythagorean Theorem states that in a right triangle $ABC$, the sum of the squares of the measures of the lengths of the legs, $a$ and $b$, equals the square of the measure of the hypotenuse $c$, or $a^2 + b^2 = c^2$. Write a two-column proof to verify that $a = \sqrt{c^2 - b^2}$. Use the Square Root Property of Equality, which states that if $a^2 = b^2$, then $a = \pm b$.

![Diagram of a right triangle](image)

**Answer:**

Given: $c^2 = a^2 + b^2$

Prove: $a = \sqrt{c^2 - b^2}$

Proof:

Statements (Reasons)
1. $a^2 + b^2 = c^2$ (Given)
2. $a^2 + b^2 - b^2 = c^2 - b^2$ (Subt. Prop.)
3. $a^2 = c^2 - b^2$ (Substitution)
4. $a = \pm \sqrt{c^2 - b^2}$ (Sq. Root Prop.)
5. $a = \sqrt{c^2 - b^2}$ (Length cannot be negative.)

An equivalence relation is any relationship that satisfies the Reflexive, Symmetric, and Transitive Properties. For real numbers, equality is one type of equivalence relation. Determine whether each relation is an equivalence relation. Explain your reasoning.

31. “is taller than”, for the set of all human beings

**Answer:**

The relation “is taller than” is not an equivalence relation because it fails the Reflexive and Symmetric properties. You cannot be taller than yourself (reflexive); if you are taller than your friend, then it does not imply that your friend is taller than you (symmetric).

33. $\neq$, for the set of real numbers

**Answer:**

The relation “$\neq$” is not an equivalence relation because it fails the Reflexive Property, since $a \neq a$ is not true.

35. $\approx$, for the set of real numbers

**Answer:**

The relation “$\approx$” is not an equivalence relation because it fails the Reflexive Property, since $a \approx a$ is not true.
5-2 Algebraic Proof

37. **CCSS SENSE-MAKING** Point $P$ is located on $AB$. The length of $AP$ is $2x + 3$, and the length of $PB$ is $\frac{3x+1}{2}$. Segment $AB$ is 10.5 units long. Draw a diagram of this situation, and prove that point $P$ is located two thirds of the way between point $A$ and point $B$.

**ANSWER:**

![Diagram](image)

Given: $AP = 2x + 3$

$PB = \frac{3x+1}{2}$

$AB = 10.5$

Prove: $\frac{AP}{AB} = \frac{2}{3}$

Proof:

**Statements (Reasons)**

1. $AP = 2x + 3$, $PB = \frac{3x+1}{2}$, $AB = 10.5$ (Given)
2. $AP + PB = AB$ (Def. of a segment)
3. $2x + 3 + \frac{3x+1}{2} = 10.5$ (Subs.)
4. $2\left(2x + 3 + \frac{3x+1}{2}\right) = 2 \cdot 10.5$ (Mult. Prop.)
5. $2\left(2x + 3 + \frac{3x+1}{2}\right) = 21$ (Subs.)
6. $2(2x) + 2(3) + 2 \left(\frac{3x+1}{2}\right) = 21$ (Dist. Prop.)
7. $4x + 6 + 3x + 1 = 21$ (Multiply)
8. $7x + 7 = 21$ (Add)
9. $7x + 7 - 7 = 21 - 7$ (Subt. Prop.)
10. $7x = 14$ (Subs.)
11. $x = 2$ (Div. Prop.)
12. $AP = 2(2) + 3$ (Subs.)
13. $AP = 4 + 3$ (Multiply)
14. $AP = 7$ (Add)
15. $\frac{AP}{AB} = \frac{7}{10.5}$ (Subs.)
16. $\frac{AP}{AB} = 0.6$ (Divide)
17. $\frac{2}{3} = 0.6$ (Known)
18. $\frac{AP}{AB} = \frac{2}{3}$ (Trans. Prop.)
5-2 Algebraic Proof

REASONING Classify each statement below as sometimes, always, or never true. Explain your reasoning.

39. If $a$ and $b$ are real numbers and $a^2 = b$, then $a = \sqrt{b}$.
   **ANSWER:**
   Sometimes; sample answer: If $a^2 = 1$ and $a = 1$, then $b = \sqrt{1}$ or 1. The statement is also true if $a = -1$ and $b = 1$. If $b = 1$, then $\sqrt{b} = 1$ since the square root of a number is nonnegative. Therefore, the statement is sometimes true.

41. WRITING IN MATH Why is it useful to have different formats that can be used when writing a proof?
   **ANSWER:**
   Sample answer: Depending on the purpose of the proof, one format may be preferable to another. For example, when writing an informal proof, you could use a paragraph proof to quickly convey your reasoning. When writing a more formal proof, a two-column proof may be preferable so that the justifications for each step are organized and easy to follow.

43. SHORT RESPONSE Find the measure of $\angle B$ when $m\angle A = 55$ and $m\angle C = 42$.
   **ANSWER:**
   $83^\circ$

45. SAT/ACT When 17 is added to $4m$, the result is $15\zeta$. Which of the following equations represents the statement above?
   A) $17 + 15\zeta = 4m$
   B) $(4m)(15\zeta) = 17$
   C) $4m - 15\zeta = 17$
   D) $17(4m) = 15\zeta$
   E) $4m + 17 = 15\zeta$
   **ANSWER:**
   E

Determine whether the following statements are always, sometimes, or never true. Explain.

47. Two obtuse angles will be supplementary.
   **ANSWER:**
   Never; the sum of the measure of two supplementary angles is 180, so two obtuse angles can never be supplementary.

49. ADVERTISING An ad for Speedy Delivery Service says *When it has to be there fast, it has to be Speedy.* Catalina needs to send a package fast. Does it follow that she should use Speedy? Explain.
   **ANSWER:**
   Yes; by the Law of Detachment
5-2 Algebraic Proof

Write the ordered pair for each point shown.

51. B

ANSWER:
(4, -3)

53. D

ANSWER:
(1, 2)

55. F

ANSWER:
(−1, −1)

Find the measurement of each segment. Assume that each figure is not drawn to scale.

57. WX

|——— 4.8 cm ———|

ANSWER:
2.4 cm